Pt 2: If "the mathematician's patterns, like the painter's and the poet's, must be beautiful" (G.H. Hardy), how might this impact the production of knowledge? Discuss with reference to mathematics and the arts.

What is beauty? According to Aristotle, it is the cooperation and symmetry of the parts in a whole, with all parts united in purpose.[1] In order to achieve unity, there is a necessity for there to be a degree of technical brilliance. According to Henri Poincare, the aim of Mathematics is to divine the hidden harmonies and relationships between numbers.[2] According to Aristotle, the aim of the Arts is to capture the essential form of things, showing their deeper meaning.[3] Furthermore, knowledge that is beautiful will have achieved the aims of its field.

While Aristotle's definition of beauty may be universal, different cultures have imposed additional conditions for what constitutes beauty. Some may sanction a certain approach, while others may find certain insights produced anathema. This essay argues that the pursuit of beauty may produce deeper insights into hidden patterns and relationships. On the other hand, the pursuit of beauty may hinder the production of knowledge, especially if cultural definitions of beauty are a factor. Despite this, knowledge produced need not be beautiful.

Firstly, without cultural influences, a more beautiful approach produces more knowledge than an ugly one. In considering a Mathematical proof, one desires to reach, *a priori*, the conclusion from starting axioms. The proof of the fundamental

[1] Will Durant, *The Story of Civilization: The Life of Greece,* (Simon and Schuster, 1935), 532–33.
[2] Henri Poincaré, "Mathematical Creation," ed. Sherwood J. B. Sugden, *Monist* 20, no. 3 (1910): 321–35, https://doi.org/10.5840/monist19102037.
[3] Aristotle, *Poetics*, trans. Stephen Halliwell (Cambridge, Mass.: Harvard University Press, 1995), i, 3.

lemma of the Langlands program, which seeks to link number theory and geometry[4], is an illustrative example. There are various approaches to proofs. The analytical approach considers all possible cases to reach its conclusion. Being applicable for just one problem, this does not show a unity between the various elements that constitute a proof and thus is not as beautiful. Furthermore, solutions with this approach are not likely to have further applications in what is not immediately obvious. Thus, less beautiful solutions, although sometimes used, produce less knowledge.

Another approach is the investigative one. Here, syllogisms flow in an unbroken chain from the axioms into the desired conclusion. This was the approach championed by Ngo Bao Chau for the fundamental lemma.[5] In this particular case, Hitchin fibration, from algebraic geometry, needed to be linked to automorphic forms, from harmonic analysis.[6] This approach holds greater beauty because there is a harmonious interplay of disparate concepts to produce the proof. The linkage of one field of Mathematics to another in connecting syllogisms would reveal as a corollary, certain deeper patterns that are unexpected. This proof helped to establish reciprocity laws for Shimura spaces, an unintended consequence.[7] This illustrates the utility of beauty in Mathematics, to achieve the latter's aims. Thus, the requirement to use a beautiful approach in Mathematics allows for the discovery of more unexpected insights and patterns, producing more knowledge.

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[4] Robert Langlands, "Publications.ias.edu," publications.ias.edu, 1979, http://publications.ias.edu/rpl/series.php?series=55.
[5] Bao Châu Ngô, "Le Lemme Fondamental Pour Les Algèbres de Lie," *Publications Mathématiques de L'IHÉS* 111, no. 1 (April 23, 2010): 1–169, https://doi.org/10.1007/s10240-010-0026-7.
[6] Bao Châu Ngô, "Fibration de Hitchin et Endoscopie," *Inventiones Mathematicae* 164, no. 2 (February 1, 2006): 399–453, https://doi.org/10.1007/s00222-005-0483-7.
[7] Mark Kisin, "Integral Models for Shimura Varieties of Abelian Type," *Journal of the American Mathematical Society* 23, no. 4 (April 21, 2010): 967–1012, https://doi.org/10.1090/s0894-0347-10-00667-3.

Secondly, cultural influences may provide direction to the pursuit of beauty and allows for insights into the human condition to be produced more easily in the Visual Arts. Here, the impact of society and culture on the definition of beauty is most prominently displayed. The Renaissance had great influence on Realism as a school of Art, which aims to portray the world realistically. This cultural vision of what constitutes beautiful art motivated artists to paint more realistically. The emotions behind important events in Christianity might be revealed, as evident in the life and work of Leonardo Da Vinci. An example of this would be Da Vinci's "The Annunciation", portraying the announcement of Mary's impending virgin birth.[8]

Figure 1: The Annunciation by Da Vinci [9]

Yet, this piece was flawed, with the Virgin Mary possessing a blank expression like a mannequin and a hand bent unnaturally over a book, not quite what the school of realism was aiming for.[10]

Da Vinci's improvements in his later art illustrates the aid cultural ideals provide for knowledge production. Said culture provides a nurturing environment for the artist to

[8] Kim H. Veltman, "Leonardo Da Vinci: A Review," *Leonardo* 41, no. 4 (2008): 381–88, https://www.jstor.org/stable/20206632.
[9] Leonardo Da Vinci, *The Annunciation*, 1472, Tempera, Oil Paint, 1472, Uffizi Gallery, https://artsandculture.google.com/asset/sAErNLFH1KFYmw
[10] Walter Isaacson, *Leonardo Da Vinci* (Simon and Schuster, 2017), 57–60.

experiment and improve. Wealthy patrons are likely to find the artist's work palatable, thus funding him, providing the security needed to experiment further, to discover new techniques. This was the case for Da Vinci, culminating in his magnum opus "The Last Supper".

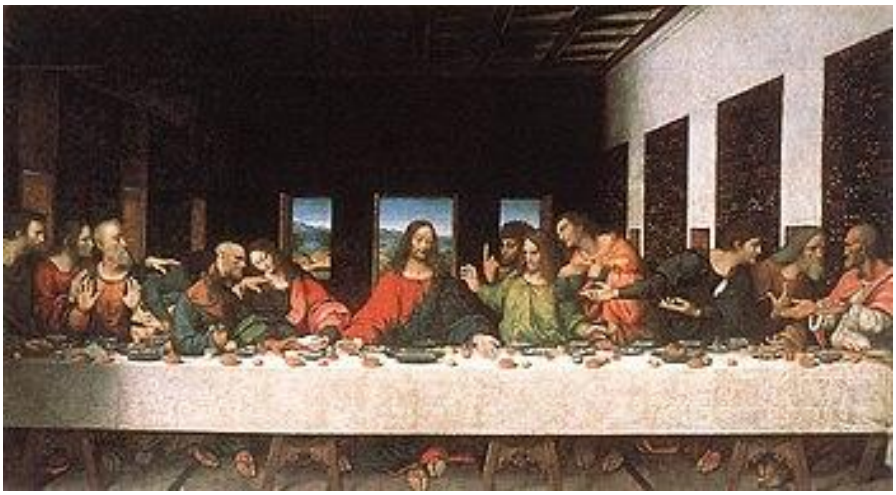

Figure 2: The Last Supper by Da Vinci [11]

Emotions are masterfully choreographed in motion, while still retaining the focus on Jesus through the use of linear perspective.[12] There is a unity of purpose here, to depict the moments of the last supper, while showing the full range of emotions in the principal actors. This art piece can thus be considered more beautiful, having better achieved the aims of realism in the Arts. Thus, cultural ideals of beauty, beyond Aristotle's basic formulation, aid the discovery of insights into human emotions in the Arts.

[11] Leonardo Da Vinci, *The Last Supper*, 1498, Tempera, 1498, Santa Maria delle Grazie, https://cenacolovinciano.org/en/
[12] Jean Paul Richter, *The Notebooks of Leonardo Da Vinci. Volume I* (New York: Dover Publications, 1970), 55.

Thirdly, cultures may allow only certain methods to be used to achieve beauty. By restricting the tools used to study Mathematics, the discovery of hidden harmonies may be hindered. Certain cultures have considered only some approaches to Mathematics beautiful, and worthy of use. For the Ancient Greeks, they allowed only the use of straight edge, compass, and the basic operations, thinking only these tools capable of producing beautiful knowledge.[13] This meant only limited areas of geometry could be studied. Some fields may not be explorable. For example, the Greeks were not able to discover the principles of algebra. Naturally, certain proofs or solutions to problems would not be obtainable, for they require techniques from unexplored fields of Mathematics. The Greeks were not able to "double the cube" for this reason.[14] This problem called for calculating the cube root of two, which was later shown to be impossible given the tools that the Ancient Greeks limited themselves to.[15]

However, broadening the understanding of what constitutes "beauty" to encompass previously ugly concepts allows for elusive knowledge to be easily produced. By using conic sections, a solution was easily achieved by Menaechmus.[16] Unfortunately, this method of solving geometry problems remained poorly developed in Greek culture, despite its apparent utility. This was due to the scorn of thinkers like Plato, who decried it as "mechanical and earthly", as compared to the compass and straightedge, which were more "noble".[17] Thus, knowledge that was insightful was produced, despite being

[13] Benjamin Bold, *Famous Problems of Geometry and How to Solve Them* (Courier Corporation, 2012).
[14] Jesper Lützen, "The Algebra of Geometric Impossibility: Descartes and Montucla on the Impossibility of the Duplication of the Cube and the Trisection of the Angle," *Centaurus* 52, no. 1 (February 2010): 4–37, https://doi.org/10.1111/j.1600-0498.2009.00160.x.
[15] Pierre Laurent Wantzel, "Recherches Sur Les Moyens de Reconnaître Si Un Problème de Géométrie Peut Se Résoudre Avec La Règle et Le Compas," *Journal de Mathématiques Pures et Appliquées* 2 (1837): 366–72.
[16] Wilbur Richard Knorr and Internet Archive, *Textual Studies in Ancient and Medieval Geometry*, *Internet Archive* (Boston : Birkhäuser, 1989), https://archive.org/details/textualstudiesin0000knor.
[17] Euclid, *Euclid's Elements (the Thirteen Books)*, 2017.

ugly. Further exploration of these insights were greatly hindered. Therefore, contemporary culture's ideas about beauty impairs the discovery of the hidden harmonies in Mathematics.

Finally, cultures may allow for only certain messages to be conveyed in order for work to be considered beautiful. In the Arts, this may slow the discovery of insights into the human condition, which "ugly" works might contain. Cultures may have ideas about the ideal human condition. The Enlightenment (1685-1815) philosophers saw the ideal man as one whose emotions are chastened by intelligence.[18] This naturally carries over into that culture's artwork, with only pieces expounding on these ideas being considered beautiful. Alexander Pope shunned irrationality in his work, particularly in the poem the "The Rape of the Lock" (1712), satirising the upper classes' frivolity.[19] To the Enlightenment man, this form of neoclassical poetry was beauty.

However, some artists may find their culture's ideas about the ideal human condition to be false. Edward Young thought that the ideal man did not exist, only *men* did, fiercely individual and jealously real. His ideas run contrary to the Enlightenment's ideals, representing evidence contrary to said ideals. Young's poem "The Complaint" (1745) is an example of such a contradiction, discussing the emotional response to death, instead of one checked by reason.[20] This is a revelation about the process of loss, directly contradicting Enlightenment ideals. To these inconvenient ideas, society reacted in a hostile manner. Both Pope [21] and Voltaire [22], champions of the Enlightenment, decried the poem as "bombast", thinking the emotional response to be

[18] William Durant and Ariel Durant, *The Story of Civilization: The Age of Voltaire,* Simon and Schuster, 1965, 178–82.
[19] Alexander Pope, *The Rape of the Lock* (London: Routledge, 1712).
[20] Edward Young, *The Complaint, or Night-Thoughts on Life, Death & Immortality*, 1745.
[21] Owen Ruffhead, *The Life of Alexander Pope*, 1769, 290–91.
[22] Hans Hertel, *Voltaire, Brandes, PH & Co.*, vol. I (Lindhardt og Ringhof, 2021), 14–17.

overblown and devoid of meaning. Clearly, they did not consider it beautiful. Yet, the emotional response is an essential component of grieving. Thus, “ugly” works are still of great importance. The ideals have restricted the exploration of knowledge in this area. From society’s hostile response, other artists may be discouraged from exploring the new ideas further. It took the end of the Enlightenment, the fading of these ideals, and the start of the Romantic period before poets started exploring human emotions in earnest, representing a loss of half a century. Thus, cultural ideas of what makes man beautiful hinders the production of knowledge that would reveal other facets of human behaviour.

One returns to the question posed at the beginning of this essay. What is beauty? As evidenced by the analytical approach, as well as the works of Menaechmus and Young, knowledge that is produced need not be beautiful by their contemporary standards. However, Romantic poets eventually deemed Young’s poems beautiful.[23] Thus, the very definition of beauty is flawed. It is not static and instead depends on the zeitgeist, the spirit of the age. Yet, spirits change, and what we consider beautiful changes with it. Works previously deemed ugly may now be beautiful, and those insights out of reach might yet be discovered.

In conclusion, since the aims of the two areas of knowledge are strikingly similar, to elucidate hidden truths, beauty’s impact on the production of knowledge is similar too. If only Aristotle’s definition of beauty is applied, deeper insights in any field of Mathematics or the Arts might be found. This was the case for Ngo’s work on the fundamental lemma. If cultures have additional conceptions about beauty, this might stimulate the discovery of deeper insights within the cultural conceptions, by

[23] James Stevens Curl, “The Tomb & the Garden: The Influence of Young’s *Night Thoughts*”, *The Georgian Group Journal* XXV (2017): 185–206.

stimulating exploration, as is the case for the Renaissance's influence on Da Vinci. However, cultural conceptions of beauty may also restrict the production of knowledge. For the ancient Greeks, this prevented them from doubling the cube. For pre-Romantic poets, their works ran contrary to the ideals of the Enlightenment and were thus suppressed.

(1600 words)

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